

# STSM Title: Natural frequencies and damping of timber structures

## Short Term Scientific Mission, COST Action FP1004

Beneficiary: Vanessa Baño, CETEMAS, Spain

**Host:** Dr. Glauco Feltrin, Structural Engineering Research Laboratory, EMPA, Swiss Federal Institute Science and Technology, Switzerland

**Period:** May 7, 2012 – May 31, 2012

Place: Dübendorf, Switzerland

Reference code: COST-STSM-FP1004-10044

## 1. Purpose of the visit

The aim of the STSM was to strengthen collaboration between EMPA and The Forest and Wood Technology Research Center, CETEMAS, from Spain, exchange ideas and deliver new important results contributing to achievement of the Action's aim. The goal was to develop the methodology to estimate the mechanical properties in timber structures from dynamic analysis and the study of the influence of the supports in the resonance frequencies in timber structures.



# 2. Description on the work carried out during the visit

## 2.1. DYNAMIC TESTS ON CONCRETE BRIDGES

**2.1.2. Objective**: The objective of the testing in concrete bridges was to know the methodology used to determine the natural frequencies and damping.

**2.2.2. Procedure**: The procedure for achieving the objective is described briefly below:

- Dynamic testing using environmental excitation
- Determination of the natural frequencies
- Excitation to the first and second natural frequency in order to estimate the associated damping







Figure 1. Dynamic test in a concrete bridge



# 2.2. DYNAMIC TESTS ON A TIMBER BEAM

**2.2.1. Objective.** The objective of the study of a simple timber beam is the study of the influence of the different supports in the natural frequencies and the estimation of the mechanical properties from dynamic testing.

**2.2.2. Definition of the beam.** A glulam beam of spruce of 140x200 mm of cross section and 4.0m long was tested, Table 1.

Table 1. Data of the timber beam

BEAM	Specie	H (%)	W (Kg)	b (mm)	h (mm)	L (m)	d (Kg/m³)
1	Glulam spruce	10	50.6	140	200	3.975	455

Where,

H, is the humidity content (%)

W, is the weight (Kg)

b, is the width of the beam (mm)

h, is the height of the beam (mm)

L, length of the beam (m)

d, is the density (Kg/m<sup>3</sup>)

# 2.2.3. Estimation of the longitudinal modulus of elasticity using Sylvatest

The dynamic modulus of elasticity of the timber beam was estimated from the wave transmission time and the density, equation 1. The time was measured for each lamella of the glulam beam and the average of velocity was calculated.

$$E_{dyn}$$
=velocity<sup>2</sup> · density

eq.1

Where,

 $E_{dyn}$ , is the dynamic modulus of elasticity estimated with the ultrasonic equipment Sylvatest-1 (GPa) Velocity, is the velocity of wave transmission (m/s) Density, is the value of density of the beam (Kg/m<sup>3</sup>)

**2.2.4. Dynamic tests in a timber beams.** The beam is excited using an impact hammer, and hitting ten times in each point of impact, and the results of accelerations are recorded in the data acquisition system. The number and definition of dynamic test are presented below.

**Accelerometers KISTLER:** measuring range=±10 g, sensitivity=500mV/g, transversal sensitivity=1.0%, resonant frequency=22.0kHz, T<sup>a</sup>=0-65<sup>o</sup>C.

Data Acquisition System: OR38 – OROS

Excitation: impact hammer, IMC



# Figure 2. TEST 1. Free-free condition.Stiff direction.



Figure 3. TEST 2. Free-free condition. Stiff direction.



Figure 4. TEST 3. Free-free condition. Soft direction.





Figure 5. TEST 4. Free-free condition. Stiff direction.



Figure 6. TEST 5. Free-free condition. Soft



Figure 7. TEST 6. Free-free condition. Stiff direction





## Figure 8. TEST 7. Free-free condition. Soft direction



Figure 9. TEST 8. Pinned-pinned. Longitudinal direction





Figure 10. TEST 9. Pinned-pinned. Stiff direction





## Figure 11. TEST 10. Pinned-pinned. Stiff direction



Figure 13. TEST 12. Pinned-pinned. Soft direction



Figure 14. TEST 13. Pinned-pinned. Soft direction





Figure 15. TEST 14. Free-free condition. Hanging horizontal







## Figure 16. TEST 15. Free-free condition. Hanging horizontal



Figure 17. TEST 16. Free-free condition. Hanging horizontal



Figure 18. TEST 17. Free-free condition. Hanging horizontal. Impact in longitudinal direction



Figure 19. TEST 18. Free-free condition. Hanging horizontal. Impact in longitudinal direction











Figure 22.TEST 21. Pinned-pinned condition. span =3.925 mm



Figure 23. TEST 22. Pinned-pinned condition. span = 3.925 mm





## Figure 24.TEST 23. Pinned-pinned condition. span =3.950 mm



Figure 25. TEST 24. Pinned-pinned condition. span =3.950 mm



Figure 26. TEST 25. Pinned-pinned condition. span =3.925 mm



Figure 27. TEST 26. Pinned-pinned condition.: span =3.950 mm





## Figure 28. TEST 27. Pinned-pinned condition.: span =3.955 mm



Figure 29.TEST 28. Pinned-pinned condition. span =3.955 mm



Figure 30. TEST 30. Pinned-pinned condition. span =3.955 mm. Environmental excitation



Figure 31. TEST 31. Pinned-pinned condition. span =3.955 mm. Environmental excitation





Figure 32. **TEST 32. Free-free condition. Hinged horizontal.** span =3.955 mm. Environmental excitation



# 2.2.5. Determination of the frequency peaks

The Matlab software was used to program a model in order to determine the average values of the peaks of frequencies corresponding to the ten impacts. The model is written in the Annex I and presents as results a graphic spectrum-frequency.

# 2.2.6. Calculation of the theoretical natural frequencies using the Euler-Bernouilli equation

The theoretical natural frequencies of the beam in a free-free condition and in a pinned-pinned condition were calculated according to the Euler-Bernouilli equation, Eq. 2.

$$f_{i-B} (Hz) = (\lambda_i^2 / 2\pi L^2) \cdot (EI/\rho)^{1/2}$$

where,

 $f_{i\mbox{-B}}$  theoretical natural frequency of the beam using the Euler-Bernouilli equation (Hz)

 $\lambda_{\text{i}},$  coefficient depending to the mode shape and the supports conditions:

 $\lambda_{i\text{-F}}$  = 4.73, 7.85, 10.99, 14.13, 17.27,... for the mode shapes: i=1, 2, 3, 4 and 5 respectively, in the free-free condition

 $\lambda_{i-P} = i \cdot \pi$ , for the mode shapes: i= 1, ..., in the pinned-pinned condition

L, length of the beam (m)

E, longitudinal modulus of elasticity  $(N/m^2)$ 

I, inertial moment (m<sup>4</sup>)

 $\rho$ , mass of the beam per length unit (Kg/m)

# 2.2.7. Estimation of the E and G from hanging experimental tests

From the experimental data of the natural frequencies in the simple beam in free-free condition and the Uniform Timoshenko Beam Theory, the values of E and G were estimated for the timber beam (Geist et al., 1998). A model was programmed in Matlab, Annex II, in order to calculate those values from the experimental and theoretical natural frequencies values.

Eq. 2



# **2.2.8.** Calculation of the theoretical natural frequencies using the Timoshenko beam equation for pinned-pinned condition

The theoretical natural frequencies of the beam in a pinned-pinned condition were calculated according to the Timoshenko beam theory (bending and shear deformation), equation, Eq. 3.

$$f_{i-P-T} = (i^2 * \pi^2 / L^2) / (2 \cdot \pi \cdot ((\rho / (EI)) + (\rho / (GA_s)) \cdot (n^2 \pi^2 / L^2))^{0.5})$$

Eq. 3

where,

 $f_{i-P-T}$ , theoretical natural frequency of the beam in a pinned-pinned condition using the Timoshenko Uniform Beam Theory (Hz)

i, mode shape

- L, length of the beam (m)
- E, longitudinal modulus of elasticity (N/m<sup>2</sup>)
- G, shear modulus  $(N/m^2)$

I, inertial moment (m<sup>4</sup>)

 $\rho,$  mass of the beam per length unit (Kg/m)

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## 2.3. Description of the main results obtained and discussion





Figure 33. Test 7



Figure 34. Test 14





Figure 35. Test 16

# 2.3.2. Results of the natural frequencies for the dynamic test in the timber beam

The results of several natural frequencies for the different test in the timber beam are presented in Table 1 The stiff direction corresponds with the beam supported on the width of the beam (b) and the soft direction, with the beam supported on the height of the beam (h). The values between f1 and f7 corresponds with the seven first natural frequencies of the beam and the test conditions represents the boundary conditions of the beam tests.



# Table 2. Natural frequencies obtained for the 32 test in the timber beam

TEST	direction	f1	f2	f3	f4	f5	f6	f7	span	TEST
		(Hz)	(Hz)	(Hz)	(Hz)	(Hz)	(Hz)	(Hz)	(m)	CONDITIONS
TEST 1	stiff		71,88	184,4	331,3	493,8	656,3	853,1		
TEST2	stiff	71,88	184,4	329,7	493,8	656,3	853,1	1036	3,975	FREE-FREE
	soft		128,1	240,6	371,9	514,1	575	846,9		HANGING VERTICAL
TEST3	stiff		184,4	329,7						Frequencies in bending
	soft	46,88	128,1	237,5	371,9	515,6				
TEST4	stiff	71,88	184,4	329,7	493,8	657,8	853,1	1036		
-	soft	48,44	128,1	239,1	370,3					
TEST5	stiff	71,88	184,4	329,7	490,6					
	soft	48,44	128,1	237,5	370,3	515,6				
TEST6	stiff	71,88	184,4	329,7	496,9	668,8	854,7	1036		
	soft	48,44	128,1	239,1	371,9	515,6				
TEST7	stiff	71,88	184,4	329,7	496,9	668,8	853,1	1033		
_	soft	48,44	128,1	239,1	370,3	515,6	670,3	845,3		
TEST8	long.	662,5	1353	2066	2713					FREE-FREE: HANGING VERT.
	long.	662,5	1325	2066	2713				3,95	Longitudinal frequencies
TEST9		х								
TEST10		Х								
TEST11		Х								PINNED-PINNED
TEST12	stiff	31,25	121,9	241,9					3,975	Frequencies in bending
	soft	20,62	80,62	206,9	316,2	413,1	546,9			
TEST13	stiff	30,62	121,9							
	soft	20,62	81,25	176,2	260	316,2	415			
TEST 14	stiff	71,25	183,7	331,2	498,7	673,7	848,7	1004		FREE-FREE: HANGING HORIZ.
TEST 15	stiff	71,25	183,7	331,2	498,7	671,2	847,5	1031	3,95	Frequencies in bending
TEST 16	soft	47,5	128,7	238,7	372,5	520	678,7	841,2		
TEST 17	long.	672,5	1062	2035						FREE-FREE: HANGING HORIZ.
TEST 18	long.	72,5	185	330	497,5	670	847,5	1032	3,95	Longitudinal freq.
TEST 19	long.		182,5	332,5	497,5	672,5	847,5	1032		
TEST 20	stiff	29,37	90,62	169,4					3,975	
TEST 21	stiff	30,62	93,12	175,6					3,925	PINNED-PINNED
TEST 22	stiff	30,62	93 <i>,</i> 65							Frequencies in bending
TEST 23	stiff	29,37	88,12	147,5					3,95	
TEST 24	soft	20,62	77,5							
TEST 25	soft	21,25	81,87						3,925	
TEST 26	soft	20,62	78,75						3,95	
TEST 27	soft	21,87	76,87						3,955	
TEST 28	soft	21,25	83,75							
TEST 30	stiff	32	97,5	182,5					3,955	
TEST 31	soft	22	76	151,5	237,5	320,5	389,5			
TEST 32	stiff	71	182,5	326						



# 2.3.3. Comparisson between the experimental and theoretical natural frequencies obtained for the free-free condition using the Euler-Bernouilli equation

The results shows that there is a low error between the experimental and the theoretical natural frequencies obtained using the Euler-Bernouilli equation for the first natural frequencies, increasing the error for the higher frequencies. Tables 3 and 4 presents the results for 8 resonance frequencies in the stiff direction and the soft direction respectively, for the beam tested hanging in the vertical position. Tables 5 and 6 presents the results for 8 resonance frequencies in the stiff direction and the soft direction and the soft direction and the soft section and the soft direction respectively, for the beam tested hanging in the horizontal position.

Table 3. Results of the experimental and theoretical natural frequencies according to Bernouilli for the beam in free-free condition in the stiff direction –Beam tested hanging in the vertical position

i	λι	f <sub>i-в</sub> (Hz)	f <sub>test-7-stiff</sub> .(Hz)	l Error l (%)
1	4,73004074	71,16	71,88	1,0
2	7,85320462	196,16	184,4	6,0
3	10,9956078	384,56	329,7	14,3
4	14,1371655	635,70	496,9	21,8
5	17,2787597	949,63	668,8	29,6
6	20,4203522	1326,34	854,7	35,6
7	23,5619449	1765,83	1036	41,3
8	26,7035376	2268,11	1211	46,6

Table 4. Results of the experimental and theoretical natural frequencies according to Bernouilli for the beam in free-free condition in the soft direction–Beam tested hanging in the vertical position

i	λί	f <sub>i-в</sub> (Hz)	f <sub>test-7-soft</sub> .(Hz)	l Error l (%)
1	4,73004074	49,81	48,8	2,0
2	7,85320462	137,32	128,1	6,7
3	10,9956078	269,19	237,5	11,8
4	14,1371655	444,99	371,9	16,4
5	17,2787597	664,74	514,1	22,7
6	20,4203522	928,44	670,3	27,8
7	23,5619449	1236,08	846,9	31,5
8	26,7035376	1587,68	1016	36,0

Table 5. Results of the experimental and theoretical natural frequencies according to Bernouilli for the beam in free-free condition in the stiff direction –Beam tested hanging in the horizontal position

i	λι	f <sub>i-B</sub> (Hz)	f <sub>test-14-stiff.</sub> (Hz)	l Error l (%)
1	4,73004074	72,07	71,25	1,1
2	7,85320462	198,66	183,7	7,5
3	10,9956078	389,44	331,2	15,0
4	14,1371655	643,77	498,7	22,5
5	17,2787597	961,68	673,7	29,9
6	20,4203522	1343,18	848,7	36,8
7	23,5619449	1788,25	1004	43,9
8	26,7035376	2296,91	1031	



i	λί	f <sub>i-B</sub> (Hz)	f <sub>test-7-soft</sub> .(Hz)	l Error l (%)
1	4,73004074	50,45	47,5	5,8
2	7,85320462	139,06	128,7	7,4
3	10,9956078	272,61	238,7	12,4
4	14,1371655	450,64	372,5	17,3
5	17,2787597	673,18	520	22,8
6	20,4203522	940,22	678,7	27,8
7	23,5619449	1251,78	841,2	32,8

Table 6. Results of the experimental and theoretical natural frequencies according to Bernouilli for the beam in free-free condition in the soft direction-Beam tested hanging in the horizontal position

# 2.3.4. Estimation of the E and G values from the experimental tests in free-free condition according to the Thimoshenko Uniform Beam Theory

The results of the E and G obtained from the model programmed in Matlab for the experiments 7, 14 and 16, in a free-free condition, are presented in the figures below.



Figure 36. Eigenvalues of Test 7 for the soft direction





Figure 37. Eigenvalues of Test 7 in the stiff direction



Figure 38. Eigenvalues of Test 14 in the stiff direction



Figure 39. Eigenvalues of Test 16 in the soft direction

From the results of natural frequencies obtained in the experimental test for the free-free condition of the beam, the values of modulus of elasticity were estimated and they are shown in Table 7.

Table 7. Estimation of E and G according to the Uniform Timoshenko Beam Theory and using the Sylvatest.

	E-Timoshenko BeamTheory			E-sylvatest	G-Timoshe	enko B.Th.	
TESTS	DIRECTION	STIFF	SOFT	LONGIT.	LONGIT.	STIFF	SOFT
	FREE-FREE CONDIT	(Mpa)	(Mpa)	(Mpa)	(Mpa)	(Mpa)	(Mpa)
test7	hanging vertical	14100	12500	х	х	890	807
tests14-16-17	hanging horizontal	14920	12920	12832	16850	835	778
	14510	12710	12832	16850	862,5	792,5	

# 2.3.4.1. Analysis of the E and G in the transversal direction

The values of E and G are slightly different in the stiff and soft direction of the cross section, being the longitudinal modulus of elasticity (*E*) and the shear modulus (*G*) higher in the stiff direction. Table 8 presents the percentage of difference between stiff and soft direction in the E and G for the free-free condition of the beam, hanging the beam in vertical and in horizontal position.

Table 8. Percentage of difference of E and G between stiff and soft direction

FREE-FREE CONDITION	E (%)	G (%)
Hanging in vertical position	11,3	9,3
Hanging in horizontal position	13,4	6,8

## 2.3.4.2. Influence of the simulation of the free-free condition in the modulus of elasticity

The influence of the type of support simulating the free-free condition are presented in Table 9. The longitudinal modulus of elasticity (E) was slightly higher when the beam was hanged in horizontal



position than in vertical. However, the shear modulus was higher when the beam was hanged in vertical position.

Table 9. Percentage of difference of E and G between hanging in vertical direction test and hanging in horizontal direction test

I		G			
Stiff (%)	Soft (%)	Stiff (%)	Soft (%)		
5,5	3,3	6,2	3,6		

# 2.3.5. Comparisson between the experimental and the theoretical natural frequencies for the pinned-pinned condition

The theoretical natural frequencies were calculated using the Euler-Bernouilli equation and the Timoshenko Beam Theory for a beam in a pinned-pinned condition (Eq. 2 and 3) and are shown below with the experimental results. The values of E and G introduced in the equation are derived from the estimation of Timoshenko Beam Theory for a free-free condition (Table 7).

Table 10. Results of the experimental and theoretical natural frequencies according to Bernouilli and Timoshenko for the beam in pinned-pinned condition in the stiff direction – Test 12

i	f <sub>i-T</sub> (Hz)	f <sub>i-B</sub> (Hz)	f <sub>n-test-12</sub>	Error iT-exp (%)	Error B-exp (%)
1	31,87	32,43	31,25	1,95	3,6
2	121,47	129,70	121,9	0,35	6,0
3	254,47	291,83	241,9	4,94	17,1

The results of the theoretical natural frequencies obtained from Timoshenko Beam Theory present a less error compared with the experimental results for the stiff direction than the results obtained from Euler-Bernouilli.

## 2.3.6. Study of the influence of the span and the supports in the natural frequencies

Due to the difficult to obtain the experimental natural frequencies in the pinned-pinned condition, several supports were designed and the influence of the span was analyzed.

Table 11. Influence of the supports and the span in the experimental natural frequencies for the pinned-pinned condition in the stiff direction

Support	Туре	Test	Excitation	span (m)	f <sub>1-stiff exp</sub> .(Hz)	f <sub>1-stiff theor</sub> .(Hz)	Error (%)
1	Solid cyilindrical	12	Impact	3.975	31.25	31.87	1.98
2	Hollow cylindrical	20	Impact	3.975	29.37	31.87	8.51
3	cylindrical+plate	30	Environmental	3.955	32.00	32.11	0.34
4	1 square-1 cylindrical	23	Impact	3.950	29.37	32.17	9.53
5	Solid squares	21	Impact	3.925	30.62	32.47	6.04

The results show that the design of the supports has influence on the results, but there is not a correlation between the span and the experimental natural frequencies. Decreases at the span, do not show higher experimental natural frequencies, due to the support design is different for each



experiment. The support 3 and the environment excitation presented the most similar results to the theoretical calculations, with an error of 0.34%

## 2.3.7. Analysis of the estimated modulus of elasticity using Sylvatest and using accelerometers

The Table 12 shows the results of time ( $\mu$ s) of ultrasonic wave propagation in the longitudinal direction through out of the timber beam. From this time and the length of the beam, the wave velocity was calculated.

Velocity (m/s) Lamella Time (µs) 625 1 6352 2 643 6174 5961 3 666 h 4 670 5925 5 660 6015 Average: 6085 b

Table 12. Results of time and velocity of ultrasonic wave throughout the longitudinal direction

The estimation of the dynamic modulus of elasticity was calculated using the equation 2 and the mean value estimated of the beam was  $E_{dyn}$ =16.85 GPa.

Tests 17, 18 and 19 present the values of the longitudinal natural frequencies of the timber beam in a free-free condition taking into account different points of impact. The peaks of the resonance frequencies for the direct measurement in the test 17 are shown in Figure 40.



Figure 40. Spectrum-frequencies graphic for the tests 17, considering the impact and the measurement in the middle lamella.

The indirect measurements showed peaks corresponding to the bending frequencies, as shows the Figure 41.





Figure 41. Spectrum-frequencies graphic for the tests 19, considering the impact point in the lamella 5 and measurement point in the lamella 1.

From a longitudinal natural frequency of 672.5 Hz in the free-free condition, the dynamic modulus of elasticity estimated was of  $E_{dyn-acc-free}$ = 12.64 GPa. From a value of 662.5 Hz of natural frequency in the pinned-pinned condition, corresponding with the test 8, the dynamic modulus of elasticity estimated was  $E_{dyn-acc-pinned}$ = 12.29 GPa.

The static modulus of elasticity was estimated from the dynamic modulus of elasticity obtained using Sylvatest, according to the equation 1.

eq. 1. (Steiger and Arnold, 2009)

The estimated static value was of  $E_{st}$ = 13.11GPa. This value is closer to the dynamic value obtained using accelerometers than the dynamic values estimated using Sylvatest.



## 2.4. Bibliography review

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2.5. Visit to timber bridges and timber structures in Switzerland





Figure 42. Covered timberbridge on the River Aare. Date: 1552



Figure 43. Thurbrücke bridge. Date: 1815



Figure 44. Bauherrschaft bridge-1992





Figure 45. Hanging bridge in Viamala. Date 2005



Figure 46. World Monument Salginatobel Bridge. Date1929



STSM Scientific Report. Vanessa Baño



Figure 47. Kapellbrucke Bridge in Luzern. Date 1365



Figure 48. 7 floors timber building in Zurich. Arq. Shigeru Ban



## 3. Future collaboration with host institution

An immediate collaboration was proposed as a continuation of the work carried out, applying the methodology used in the simple beam on a timber bridge, with an analysis of results together.

Other future collaborations were raised in the field of structural engineering of hardwoods.

## 4. Projected publications/articles resulting or to result from the STSM

The work done during the STSM visit and further planned tests at EMPA and CETEMAS will lead the publication of the one or two papers in the field of vibration in timber structures:

- Behaviour analysis of the supports on timber structures using the experimental modal analysis
- Study of the modal parameters of a timber footbridge

## 5. Confirmation by the host institute of the successful execution of the mission

Attached in a separate file

## 6. Other comments

Besides the benefits mentioned above, the STSM has allowed establishing the basis for future joint proposals for European grants.



#### **ANNEX I**

#### 1. EXCITATION USING AN IMPACT HAMMER

```
%TEST 5
```

```
functionaverageSpectrum
closeall
% define the file name
dataDir = 'C:\Users\bgm116\Documents\MATLAB\vanessa-beam1\data\';
tstNme = 'test5';
nfft = 2048;
Pxx2=[]; % average
Pxx3=[]; % average
% loop over the records
forcnt=0:9
filNme = [dataDir,tstNme,'_Rec',sprintf('%d',cnt),'.mat'];
load(filNme)
dt = Track1_X_Resolution; % time intervaal
fs = 1/dt; % sampling rate
    [pxx2,f] = pwelch(Track2,window(@rectwin,nfft),nfft/2,nfft,1/dt); %
compute the spectrum stiff direction
    [pxx3,f] = pwelch(Track3,window(@rectwin,nfft),nfft/2,nfft,1/dt); %
compute the spectrum soft direction
ifcnt==0
        Pxx2 = pxx2;
        Pxx3 = pxx3;
else
        Pxx2 = Pxx2 + pxx2;
        Pxx3 = Pxx3 + pxx3;
end
end
Pxx2 = Pxx2/10;
Pxx3 = Pxx3/10;
fp3 = [48.44,128.1,237.5,370.3,515.6]; % natural frequencies soft
direction
fp2 = [71.88,184.4,329.7,490.6]; % natural frequencies stiff direction
% plotting
semilogy(f,Pxx2,'Color','blue'); % plots average spectrum stiff direction
line(f,Pxx3,'Color','red'); % plots average spectrum soft direction
yRng = [1.0e-13 1.0e-3];
fori=1:length(fp2)
line([1 1]*fp2(i),yRng,'Color','blue','LineStyle','--')
end
fori=1:length(fp3)
line([1 1]*fp3(i),yRng,'Color','red','LineStyle','--')
end
xlim([0 1400])
xlabel('frequency [Hz]')
ylabel('spectrum')
```



**ANNEX I** 

## 2. ENVIRONMENTAL EXCITATION

functionaverageTFOneFile

```
closeall
dataDir = 'C:\Projekte\vanessa\data\';
tstNme = 'test11_Rec0';
nfft = 2048;
filNme = [dataDir,tstNme,'.mat'];
load(filNme)
dt = Track1_X_Resolution;
fs = 1/dt;
[Pxx2,f] =
tfestimate(Track1,Track2,window(@rectwin,nfft),nfft/2,nfft,1/dt);
[Pxx3,f] =
tfestimate(Track1,Track3,window(@rectwin,nfft),nfft/2,nfft,1/dt);
fp2 = [30];
fp3 = [20];
semilogy(f,abs(Pxx2),'Color','red');
line(f,abs(Pxx3),'Color','blue')
yRng = get(gca,'YLim');
fori=1:length(fp2)
line([1 1]*fp2(i),yRng,'Color','red','LineStyle','--')
end
fori=1:length(fp3)
line([1 1]*fp3(i),yRng,'Color','blue','LineStyle','--')
end
xlim([0 400])
xlabel('frequency [Hz]')
ylabel('transferfunction')
```



**ANNEX II** 

```
1. EIGENVALUES TEST 7-STIFF DIRECTION
```

```
functionfindEigTIsBeam
```

```
% estimation of modulus of elasticity and
clc
closeall
fig = figure(1);
set(0,'DefaultFigurePaperType','A4');
set(fig,'DefaultAxesColorOrder',[0 0 1;0 0.7 0; 0 0. 1;1 0 1]);
set(fig,'Units','centimeters','PaperUnits','centimeters');
set(fig,'DefaultAxesFontName','Times');
set(fig,'DefaultAxesFontSize',16);
set(fig,'DefaultAxesLineWidth',1);
set(fig,'DefaultAxesColor','none');
set(fig,'DefaultAxesXColor','black');
set(fig,'DefaultAxesYColor','black');
set(fig,'DefaultAxesBox','on');
set(fig, 'Position', [10 5 40 12]);
% parameters
as = 0.14;
bs = 0.2;
L = 3.975;
As = as*bs;
Is = as*bs^3/12; % stiff
% Is = as^3*bs/12; % soft
Em = 13.*1.e9;
Gm = Em/15;
nu = 0.2;
k = 10*(1+nu)/(12+11*nu);
% k = 1.0e9;
rho = 450;
% natural frequencies
p0 = [71.8,184.4,329.7,496.9,668.8,853.1,1033,1211]; % stiff
% p0 = [48.4,128.1,239.1,370.3,515.6,670.3,845.3,1017,1181,1348]; % soft
fSup = 1400;
pp = [1:1:fSup];
yy = zeros(fSup,1);
z0 = [Em, Gm];
z = fminsearch(@optPar,z0);
functioneps = optPar(z)
Em = z(1);
Gm = z(2);
        x = zeros(1, length(p0));
fori=1:length(p0);
x(i) = fzero(@eigFunc,p0(i)*2*pi)/2/pi;
end
fori=1:length(pp)
```

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**ANNEX II** 

```
yy(i) = eigFunc(pp(i)*2*pi);
end
eps = sum((p0-x).^{2});
function y = eigFunc(p)
            r = sqrt(Is/As)/L;
            s = sqrt((Em*Is)/(k*As*Gm))/L;
            b = sqrt(rho*As/Em/Is)*L^2*p;
            a = sqrt((r<sup>2</sup>+s<sup>2</sup>)/2-sqrt(((r<sup>2</sup>-s<sup>2</sup>)/2)<sup>2</sup>+(1/b)<sup>2</sup>)); % Eq. 10
            B = sqrt((r^2+s^2)/2+sqrt(((r^2-s^2)/2)^2+(1/b)^2)); \& Eq. 11
            y = 2*(1-\cos(b*a)*\cos(b*B))+b/sqrt(b^2*r^2*s^2-
1)*(b^2*r^2*(r^2-s^2)^2+3*r^2-s^2)*sin(b*a)*sin(b*B); % Eq. 13
end
end
subplot(1,3,1)
plot(pp,yy)
line([1 fSup],[0 0],'Color','black')
line(x,zeros(length(x),1),'Marker','o','LineStyle','none','Color','red')
ylim([-1 1]*fSup)
xlabel('frequency [Hz]')
ylabel('eigenvalue equation')
title('zero crossing check')
subplot(1,3,2)
plot([1:1:length(p0)],p0,'Marker','o','LineStyle','none')
line([1:1:length(x)],x,'Marker','d','LineStyle','none','Color','red')
xlabel('mode # [Hz]')
ylabel('frequency [Hz]')
title('comparison measured/computed')
subplot(1,3,3)
plot([1:1:length(p0)],x./p0,'Marker','o','LineStyle','none')
ylim(1+[-1 1]*0.05)
set(gca,'YTick',1+[-5:1:5]*0.01)
xlabel('mode # [Hz]')
ylabel('frequency ratio')
title('comparison measured/computed')
fprintf('\n\n Elastic modulus: %g [MPa]',z(1)/1.e6)
fprintf('\n Shear modulus: %g [MPa]',z(2)/1.e6)
fprintf('\n E/G: %g',z(1)/z(2))
fprintf(' \ n \ )
```

end